THE EFFECTS OF GEOMETRY AND MATERIAL PROPERTIES ON THE COEFFICIENT OF RESTITUTION

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Abstract—The coefficient of restitution is one of the major parameters that characterize a collision process. The experimental evaluation requires a series of distinctive aspects. The paper presents the influence produced by the materials of the colliding bodies and their geometries on the coefficient of restitution.

Keywords—coefficient of restitution, curvature radius, experimental values

I. INTRODUCTION

THERE are two main manners in describing a collision phenomenon: the first one considers the impact as an instantaneous process, [1] and the other one regards the impact as occurring during a finite time period, [2], the kinematical and dynamical parameters continuously varying all over. The models from both classes evolved in time and therefore nowadays there are available a variety of methods in approaching an impact phenomenon. As a general characteristic for all the models, a parameter named coefficient of restitution COR is used. The coefficient of restitution is, in principle, the ratio between the values of a parameter measured at the moment when the impact ends t_f, and at the moment when the first point of the two bodies come into contact, t_i, respectively. Between these two moments, the time t_c must be noticed, representing the instant when the maximum approach between the two bodies is reached.
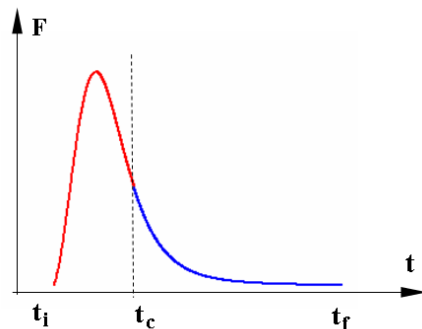

Fig. 1. Force versus time plot for a general impact

For an actual collision process it is possible that the maximum impact force does not arise at the instant t_c.

The variation in time of the impact force in the case of centric collision of two spheres is presented in Fig. 1. The kinematical manner, due to Newton, [3], is simplest definition method of coefficient of restitution, as the ratio between the normal components of the relative velocities corresponding to moments t_f and t_i respectively.

The values of the parameters in the present work are onward denoted by (') and (") respectively. Accordingly:

$$e_{Newton} = \frac{v_n''}{v_n'} \qquad (1)$$

The limits of this definition were pointed out by Kane [4], who considers a double pendulum impacting an horizontal rough plane. Dry friction is considered for all the joints of the system. By convenient selection of initial position of pendulum and the values of coefficient of friction, Kane [4] demonstrates that by using (1) for the coefficient of restitution, a paradoxical situation is attained as the final kinetic energy is greater than the initial one. To surpass this difficulty, the definition of the coefficient of restitution by means of dynamic parameters, using the normal components of percussions is required. The percussion is defined as:

$$\overline{P} = \int_{t_i}^{t} \overline{F}\,dt \qquad (2)$$

The coefficient of restitution is defined, according to Poisson, [5]:

$$e_{Poisson} = \frac{\int_{t_c}^{t_f} \overline{F} \cdot \overline{n}\,dt}{\int_{t_o}^{t_c} \overline{F} \cdot \overline{n}\,dt} \qquad (3)$$

II. THEORETICAL ASPECTS

A graphical technique was proposed by Routh, [6], for the study of collision in the case of plane systems, namely *the plane of percussions method*, manner that proved to be above all straightforward and efficient.
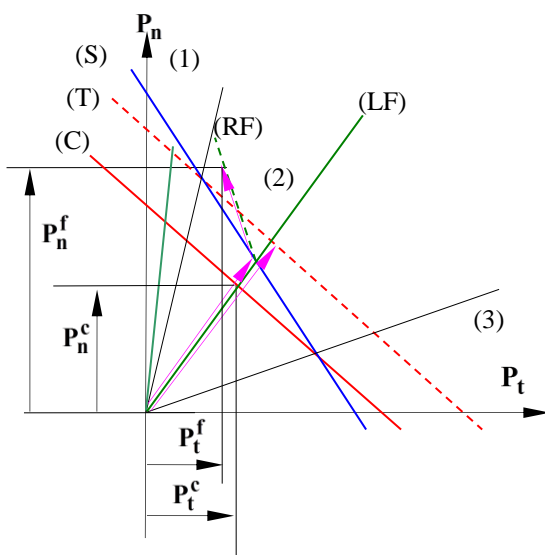
Fig. 2. The plane of percussions proposed by Routh, [6]

The percussions plane described by Routh is the P_t–P_n plane. The following straight lines can be observed in this plane:

*1) (*C*)- the maximum compression straight line;*
2) (T) - the straight line on which the end of impact occurs, according to Newton;
3) (S) - the stiction straight line, on which the sliding ends;
4) (LF) - the limit friction straight line $P_t = \mu P_n$ and
5) (RF)- reverse sliding straight line, the line on which the sliding inverses its direction, is symmetrically tilted about vertical line, similarly to the limit friction line.

The straight lines connecting the origin to the points of intersection between the straight lines (S) and (T), and (C) and (S) respectively, generate in the Routh plane three domains denoted by (1), (2) and (3). A point from Routh plane corresponds to an instant during collision characterised by percussions P_t, P_n. At the initial moment, the characteristic point is placed in origin. Within the assumption of initial sliding presence, the point will track the limit friction straight line, the normal percussion increasing monotonically from zero to the value corresponding to the impact ending. As an example, one considers the characteristic point of collision positioned in the domain (2), till meets the straight line (C), crosses it, reaches the straight line (S) and from this position, according to Newton, moves on the straight line (LF) to the line (T) where the collision finishes. In conformity with Poisson, after the point reaches the stiction line it will continue to move, either on it or on the (RF) line, depending on which one is closer to the vertical.

According to [7], the collision ends when the following relation is fulfilled:

$$P_f^n = (1 + e_{Poisson}) P_c^n \qquad (4)$$

After finding the final point of collision, the normal component of final percussion can be evaluated. It

must be noticed that *only in the region (1), characterised by the lack of sliding, the final points of contact coincide for both hypotheses.*

The emerging conclusion is that the coefficient of restitution should be evaluated by experimental tests in which the sliding is excluded.

III. METHODOLOGY AND TEST RIG

Considering the aspects revealed as theoretical fundaments, the coefficient of restitution was found using a simple method. A steel ball falls free, from a height h against a prismatic metallic body. Consequent to collision, the ball rebounds vertically and after a time period Δt will collide once more the prism's surface. The flight time of the ball is measured by recording the acoustic signal and transducing it into a visual signal. The impact instants are very well delineated by the shock waves, [8], as seen in Fig. 3.

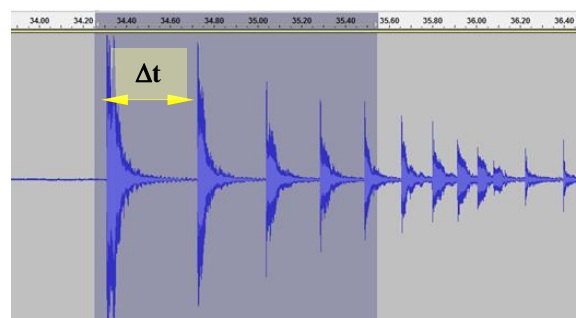The characteristic waves of successive collisions can be evidently observed from Fig. 3.

Fig. 3. The image of acoustic signal of multiple collisions

The coefficient of restitution can be obtained knowing the launching height h and the flight period Δt using the relation:

$$e = \frac{v''}{v'} = \frac{g\Delta t / 2}{\sqrt{2gh}} \qquad (5)$$
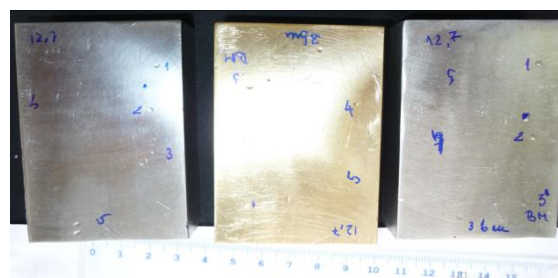
where g is the acceleration due to gravity.

Fig. 4. Prisms with the imprints after impact

The experiments were carried out using two bearing balls with diameters $d_1 = 12.7$(mm) and $d_2 = 17.45$(mm). Three prisms were used, made of steel, aluminium and brass. The balls were launched from heights within the range: h={1.272;1.035;0.853;0.442;0.278}(m) .

As Goldsmith underlines, [7], the coefficient of

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restitution is not constant, but depends on velocity. The dependence can be considered linear for velocities overmatching a definite value.

The main aim of the work is obtaining diagrams for the variation of coefficient of restitution COR, with velocity and the geometry of the respective material. These graphs are extremely useful in dynamical analysis of multibody systems. As example, in Flores's equation, [9]:

$$F = Kx^n \left[1 + \frac{8(1-e)}{5e} \frac{v}{v'_0} \right] \qquad (6)$$

the number of initial parameters characteristic to the model diminishes when certain dependencies between these parameters (geometry and material, K, velocity v_0 and coefficient of restitution, e) are known.

The values of coefficient of restitution for the three materials used are presented in TABLE I, where Case 1 corresponds to the $d_1 = 12.7$(mm) diameter ball and Case 2, to the $d_2 = 17.45$(mm) diameter, respectively.

TABLE I
EXPERIMENTAL VALUES OF COEFFICIENT OF RESTITUTION

Prism material					
Aluminium		Steel		Bronze	
Case 1	Case 2	Case 1	Case 2	Case 1	Case 2
0.284	0.209	0.436	0.397	0.394	0.354
0.316	0.249	0.480	0.432	0.429	0.393
0.285	0.193	0.490	0.427	0.421	0.385
0.353	0.268	0.506	0.425	0.471	0.428
0.359	0.286	0.529	0.483	0.483	0.428

For the three materials, the dependencies of coefficient of restitution on dimensionless velocity are presented in Figs. 5, 6 and 7, for collisions with the two balls, respectively together with the interpolation straight lines of the data. The dimensionless velocity was obtained by the ration between the current value of velocity and the maximum impact velocity. For all tests, the results for d_1 ball are plotted in red and for the d_2 ball are plotted in blue, respectively, in Figs. 5-9. The solid lines are used for experimental data while the dashed-dotted lines are used for interpolation lines. The experimental data for all tests and the interpolation lines are presented comparatively in Fig. 8 and Fig. 9, respectively.

The interpolation straight lines have negative slope and thus confirm the decreasing tendency of coefficient of restitution with increasing relative impact velocity. Starting from energy hypothesis, Tabor, [10], obtains a relationship between the initial velocity v' and final velocity v'' for the impact between a ball and a fixed anvil:

$$v'' = k(v'^2 - \frac{3}{8} v''^2)^{3/8} \qquad (7)$$
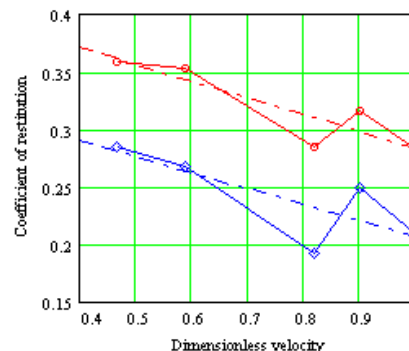
where k is a constant.

Fig. 5. The variation of coefficient of restitution with velocity for the aluminium prism
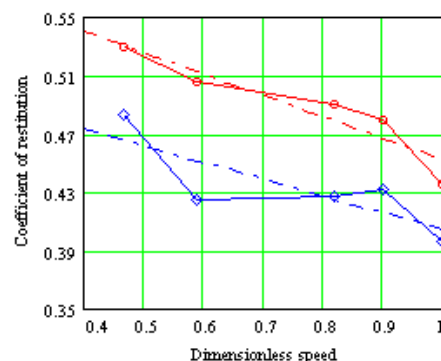

Fig. 6. The variation of coefficient of restitution with velocity for the steel prism
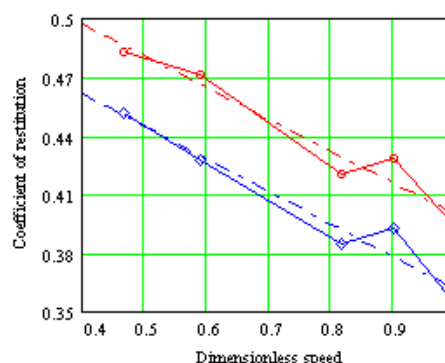

Fig. 7. The variation of coefficient of restitution with velocity for the bronze prism
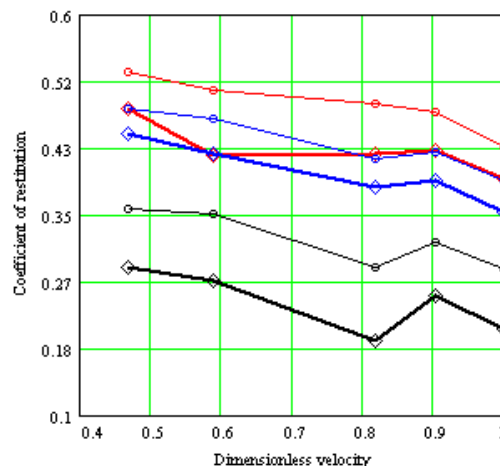

Fig. 8. The variation of coefficient of restitution with velocity for all experiments

$m_{OL1} = -0.146$

$m_{OL2} = -0.115$

$m_{BZ1} = -0.162$

$m_{BZ2} = -0.167$

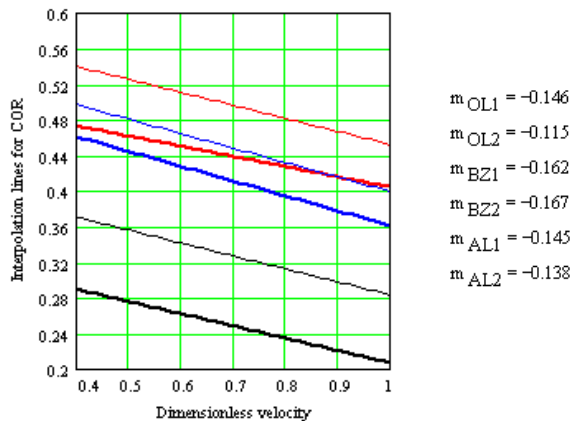$m_{AL1} = -0.145$

$m_{AL2} = -0.138$

Fig. 9. Interpolation lines and the slopes values, respectively, for the three materials and two ball radii

Based on this relation, one can prove that for great values of initial impact velocity, the variation of coefficient of restitution can be approximated to a linear one. For reduced collision velocities, this dependency is not linear any longer. A laser scan analysis of successive impact imprints produced by a steel ball dropped onto a plane surface is presented in Fig. 10. It is observed that for collisions with impact velocity v'<1(m/s) the indentation profile differs slightly one from another and the depth of the plastic imprint and the surface irregularities have the same magnitude order.
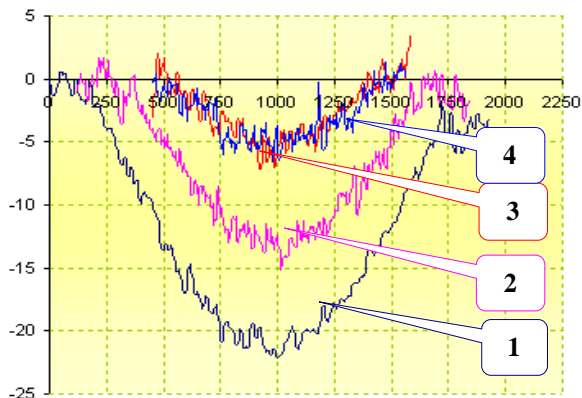

Fig. 10. Plastic indentation profiles for four successive collisions
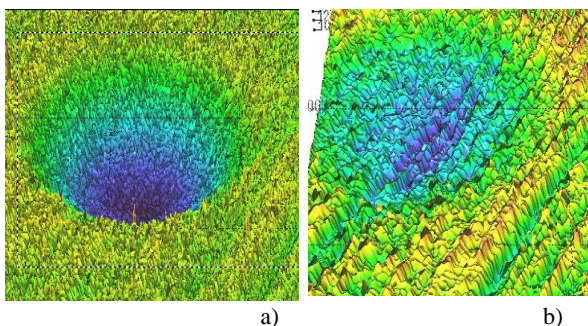

a)   b)

Fig. 11. Laser scanned images of first (a) and fourth (b) successive collision imprints

The spatial scanned images for the first and the fourth indentation, Fig. 11, show that for the first imprint it is obvious the shape of the indenter but for the last mark, only axial symmetry is visible. For the fourth imprint, the contact cannot be regarded as a sphere-plane one, but as a contact between two random surfaces and the impact has only local effects, the asperities tips acquire the whole impact energy; a consequence at bodies volume scale is not noticed. The conclusion arising is that for analysis of impact at low velocities, the impacting surfaces should be extremely fine polished.

IV. CONCLUSIONS

The coefficient of restitution COR presents the greater values for steel, than follows bronze and next aluminium. For all cases, from the interpolation lines, for the smaller diameter ball, at the same impact velocity, the values of coefficient of restitution are greater. For the same material, the slopes of interpolation lines are practically the same.

For the steel prisms, there are observed the smallest variations with velocity of the coefficient of restitution. For the prisms made of a certain material, the variations with velocity of COR illustrate approximately the same gradient. There were obtained dependencies of coefficient of restitution as function of velocity having linear form, easy to operate with. These interpolation functions are extremely valuable in the dynamic multibody analysis when between two elements multiple collisions happen and the coefficient of restitution cannot be evaluated for each one collision. Thus, in a dynamic multibody contact problem, the initial velocity and the coefficient of restitution are no longer independent parameters, but resuming one of them, usually the velocity, is satisfactory.

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